

	Columban J. Life Sci.	Vol. 17	No. 1	1-9	2015
---	------------------------------	---------	-------	-----	------

MINIMUM THICKNESS REQUIREMENT OF CFRP IN BUILT- UP TIMBER COLUMNS IN COMPRESSION AND FLEXURE

Abejide, O. S* and Abdulkarim, A.

Department of Civil Engineering, Ahmadu Bello University, Zaria. NIGERIA.

Email: Abeji.os@gmail.com

Received 12 October 2014,

Accepted 28 December 2014

ABSTRACT

Stochastic evaluation of the minimum thickness requirement of CFRP in built-up timber columns in compression and flexure is presented. It is carried out using the Box complex method for complex constraint problems. The critical load is obtained for pin ended columns made of circular built-up glulam timber which is also externally wrapped with CFRP. The critical load is used to find the constraint equation which is the difference between the critical load and the safe load of glulam for the four strength classes employed. The safe load is obtained from the relevant Eurocode. The constraint equation and critical loads are then considered using the Box method with the critical load as the objective function and variables such as: column diameters, CFRP thicknesses, and lengths of columns. The method is applied by varying one parameter at a time, while leaving the others constant. When CFRP thickness is varied, it is observed that for some increases in the CFRP thicknesses there are increases in the minimum function which is the measure of the strength or capacity of the column. Therefore, it is concluded herein, that increases in CFRP thicknesses are recommended up to an optimum value depending on the lengths or spans in order to strengthen structural members like columns, beams, etc., as long as delamination or de-bonding does not occur; and the Box method can be safely used for analysing structural members. Also, it is clear that relevant Eurocodes prove to be suitable and convenient design criteria.

Keywords: glulam timber columns, flexure, compression, CFRP, optimization, wrap thickness.

INTRODUCTION

In many applications, timber columns can be built-up from smaller dimensional lumber and which may be connected with fasteners or adhesives. Such built-up columns may be solid or hollow cross sections. Traditional rectangular hollow columns are a specific type of built-up columns in which four component members are placed around the periphery of the section, leaving a hollow core in the middle. These columns are typically assembled with mechanical fasteners (nails, ring connectors, etc.). Their form does not lend itself easily to the use of adhesives (Harries and Petrou, 1999). An alternative built-up column arrangement has been introduced Cox, 2000). This arrangement of dimensional lumber lends itself to automated fabrication using adhesives relatively easily. This arrangement however, is not bi-axially symmetric as it is the typical arrangement. It has been hypothesized, therefore, that the Cox (2000) built-up columns may have a strong and weak axis (Harries

and Petrou, 1999). Furthermore, the lack of mechanical connectors requires that the adhesive transfer all shear forces in the section directly. This requirement implies a uniform adhesive line to be maintained. Hollow sections derive their efficiency from placing the material around the periphery of the section where it is most efficient in resisting buckling loads. A hollow section has a greater radius of gyration (measure of the ratio between moment of inertia and cross sectional area) than a solid section having the same cross sectional area and is thus inherently more stable. Unfortunately very little guidance is available for the design of hollow rectangular columns (Harries and Petrou, 1999). Available standards consider composite (or built-up) members only in as far as providing sufficient connections between the component timbers. Essentially, the design of built-up sections amounts to selecting an appropriate spacing for the connection

hardware used (Harries and Petrou, 1999). Built-up sections fabricated using adhesives are often left to the designers to determine appropriate methods of treating the design of the columns. It has been stated (NDS, 191) that the glue in the joint is always stronger than the timber itself. Thus for built-up columns, individual laminations of mechanically laminated members shall be designed in accordance with relevant clauses relating to solid columns. Hence, it is felt that this guidance may be unduly harsh if applied to built-up hollow columns. Also, there is very little literature available pertaining to timber rectangular hollow or box columns. The available literature mostly discusses mechanically connected box columns and the role that the mechanical connections play in the behaviour of the columns. However, Dyer (1992) presents a model for determining the allowable axial stress in a box column of varying length and some experimental verification. The model is directed towards optimization of mechanical connectors (nails, in this case) and considers a column geometry as an indication of the efficiency of a box column; and reported that a 3m column having excellent connection details has an axial stress carrying capacity approximately 1.8 times that of a solid column having the same sectional area. Other researchers (Neubauer, 1972; Kinzey, 1951) have also addressed mechanically connected rectangular hollow or box columns. The apparent advantages of the built-up columns produced by Cox (2000) are as follows: (i) efficient use of material allowing significantly greater loads to be carried and higher allowable stresses to be used; (ii) improved quality control stemming from the use of smaller dimensional lumber sections; (iii) ability to use an automated fabrication procedure which would not be suitable for traditional box columns; and (iv) reduced cost-to-column capacity ratio.

The essence of this study is to determine the minimum thickness requirement of CFRP reinforced timber columns in compression and flexure. This work is considered due to the characteristic advantages CFRP has over other structural engineering materials such as steel (Michael, 2006). But the compressive strength of CFRP is low compared to its tensile strength even though it is subjected to brittle failure, hence the need for optimization in a stochastic or probabilistic environment, in order to determine the optimum option for structural safety and economy.

This work aims at predicting and optimizing the required minimum thickness of CFRP in built-up

timber columns in compression and flexure for practical applications. Thus, the objectives are as follows: (i) determine the minimum requirement of CFRP thickness in built-up timber columns; (ii) examine the efficiency of CFRP built-up timber column sections in compression and bending; and (iii) predict the buckling of CFRP columns using Box's complex method for constrained problems. However, the scope herein, is to optimise the thickness and design of CFRP wrapped timber built-up columns when subjected to bending and axial compression only in service. But the limitations considered are as follows: (i) investigation and research is currently on-going on using CFRP laminates both for retrofitting and as an alternative to steel as a reinforcing or prestressing material. Cost remains an issue and long term durability questions still remain. Some are about the brittle nature of CFRP, in contrast to the durability of steel; (ii) when CFRP bars are subjected to load in tension, they exhibit linear stress-strain behaviour up to rupture; (iii) there have been attempts to improve the ductility of CFRP with little or no success; and (iv) structural limitations of CFRP is that it lacks fatigue endurance limit

BACKGROUND OF WORK

CFRP confinement as a strengthening system has been found to increase the load capacity of structures. The uncertainty of the mechanical properties of timber can be dissipated through the use of CFRP wraps. The laminates are carbon fibre reinforced polymers. The fibres, which are highly anisotropic, (Pearce, 1970) provide the stiffness and strength of the system, while the polymer matrix holds the fibres in place. As timber does not have equal properties in all directions, confinement with CFRP can help to mitigate the random character of wood (Kasal and Heiduschke, 2010). Fibres are used from a mass of material, a significant increase in strength and decrease in brittleness occurring when material undergoes an extrusion-like process (Heslehurst, 2008). The bond of the confinement of timber with CFRP is of great significance. The epoxy matrix displays excellent adhesion and strength, forcing the individual and flexible fibres to cooperate in the same direction, transfer loads between the fibres and protect the fibres from environmental factors (Pearce, 1970). The epoxy matrix is cured by the addition of a hardener, the mix forming a chemical bond. When cured, the once flexible and workable fibres become very stiff. This is important for compressive loads and the avoidance of buckling. Strengthening of

structures is a requirement for rehabilitation of structures in order to increase its structural capability. Compressive strength, ductility and environmental protection are three of the factors considered to be benefited from confinement. The benefits vary depending on wrap number and orientation, and produce differing failure mode from the reference samples (Zhang, Song, Gu & Tang 2012, Heiduschke & Haller 2012, Kasal & Heiduschke, 2010). CFRP plates in flexure, buckling and the thickness optimization of CFRP wrapped built-up timber columns in compression and bending can offer a very good advantage as structural members. The compressive forces are jointly transferred to the timber and to the carbon fibre reinforcement polymers, thereby increasing the strength of the column. Tests have shown that with the combination of different orientations and types of wraps, as well as number of layers, different benefits can be found including increase in strength, ductility and stiffness (Zhang et al., 2012; Heiduschke and Haller, 2012; Kasal and Heiduschke, 2010). Dissimilar failure modes were also observed in comparison to the reference specimens. The notion of CFRP's for rehabilitation of timber structures has not been examined to satisfied extent. The benefit of confining timber has mostly been documented from experimental research. Some of the advantages of using CFRP-confinement for reinforcement are that it is durable, corrosion resistant, easy to use and transport due to its high strength to weight ratio and is more flexible than steel. In a circular column the axial capacity may be increased, enhancing the compressive strength. Comparably to steel plate bonding, an alternative strengthening system, CFRP confinement requires less man power and scaffolding as it is a light material, thereby, saving time and money.

There are so many advantages derived from wrapping timber with CFRP. Some of the main advantages of CFRP wrapping of timber are (Najm et al., 2007; Heiduschke and Haller, 2012; Zhang et al., 2012; Jimenez et al., 2011; Tsakania and Mouzakis, 2010; Webber and Yao, 2001): (i) increase in compressive strength; (ii) protection from environmental degradation; (iii) restoration of historical buildings or structures; (iv) successful application in retrofitting of structures; and (v) can be designed or manufactured to meet specific mechanical properties. Emerson (2004) found that transverse reinforcement increased the strength of the column so that it exceeded that of the design value of the column. Additional compressive and bending strength was

provided by longitudinal reinforcement. In both of these cases, full wraps were utilised, which has shown to produce the most promising results. Longitudinal cracks in timber are a common concern and reduce compressive strength in columns. Cai et al (2012) found that with wraps 50 mm wide and 115 mm apart that eccentric load capacity could be increased, and that the CFRP could contain further crack opening and confine local ruptures. Najm et al (2007) compared the behaviour of full wraps and spirals, finding that the full wrapping had greater benefits than those with spiral reinforcement. This coincides with research conducted by Zhang et al (2012), a main variable being CFRP spacing and width along the length of the timber columns. The benefits increased with smaller spacing between wraps and an increase in the width of the sheets. The number of layers also had an effect with Zhang et al (2012) finding that results became stable after three layers of CFRP were applied. The use of three layers produced the best results for Najm et al (2007) and Song et al (2010). They found that tests fully confined with CFRP showed an increase in load capacity as more layers were added. A favourable failure mode occurs when deformation is observed before failure. A sudden failure can cause sudden collapse without warning, potentially resulting in the loss of lives. The failure modes observed by Heiduschke and Haller (2012) of reinforced tubes demonstrated ductile behaviour, the wood fibres crushing parallel to the grain, in conjunction with local buckling. Song et al (2010) found that two types of failure modes were prevalent, significant changes in the columns occurring in both types around 70% of the maximum loads. For the first type of failure, compression wrinkles became apparent at mid-height of the specimens which then propagated until maximum load, while in the second type, significant crushing deformation occurred near the ends of the cylinders, the deformations propagating until maximum load.

Use of Timber and CFRP in Construction of Columns

Timber

Timber as a construction material continues to be a popular choice in modern infrastructure, being the oldest structural material, it remains useful for construction in a world of diminishing resources and is a major component in most historic buildings. Timber has many positive structural and aesthetic properties, and is an energy-efficient and renewable resource (Lloyd and Singh, 1992). There are many

timber structures in Australia, most notably bridges with their main structural component being timber piles. It is estimated that 12000 timber bridges are still in service in Australia today (Kluft, 2011), many of which are under constant repair as a result of a requirement to increase the strength in the bridges. Deterioration of strength may be attributed to aging, or mechanical and environmental loadings. Strict environmental laws in relation to the use of toxic treatments for wood have led to higher maintenance costs for timber, due to environmental degradation concerns. Lack of maintenance and design load changes affect functionality. Design changes include increased traffic flow, requirement for more lanes, width of maximum load and spacing between vehicles. (Nezamian et al., 2004) Due to these requirements, either entire structures or components need to be replaced, or rehabilitation is necessary.

Projects are governed by economics, and so the cost must be considered as a vital part of engineering management. The concept of rehabilitation to increase a structure's suitability should be considered as an alternative to complete reconstruction. Constructing a new bridge or undertaking large reconstruction can be uneconomical and timely due to jacking up of the structure while repairs occur. The replacement of structural components would generally require the use of hardwood which is generally stronger than softwood, due to its superior density. This is typically an expensive process (Emerson, 2004), also creating sustainability issues due to the difficulty of obtaining old growth mature forest sawn timber (Walter, 1996). For softwood to be used in an outside environment, it generally has to be treated with toxic preservatives to protect it against fungi and insect pests. Although considered weaker, softwood timber forests are sustainable due to the speed at which the trees grow (Walter, 1996). If the softwood timber were able to be sufficiently protected, it could make for a viable option environmentally. Due to these concerns, bridge restoration using alternative construction methods must be considered. Carbon Fibre Reinforcement Polymers (CFRP) have the capability to strengthen timber components in compression. Limited research has been conducted on layers of CFRP as a protective method from environmental degradation, but some promising results have been demonstrated (Pearce, 1970).

In most bridges the main structural components are timber piles and beams. Timber is another name for wood whether still standing in the form of trees or felled and turned in to boards for construction. The

various types of wood are soft and hard woods. The properties of timber for structural applications include; density, specific gravity, moisture content, durability, amenability to preservatives, growth pattern, etc.

Fibre reinforced plastics (FRP)

Composite is defined as a mechanically separable combination of two or more component materials, different at the molecular level, mixed purposefully in order to obtain a new material with optimal properties, other than the properties of the components. The combination of reinforcing steel and concrete has been the basis for a number of structural systems used for construction for the last century. The new class of composite materials, gradually gaining acceptance from civil engineers, both for the rehabilitation of existing structures and for the construction of new facilities, are Fibre Reinforced Polymer composites, primarily developed for the aerospace and defence structures (Potyrala, 2011). A fibre is a material made into a long filament. A single fibre usually has a diameter up to 15 μm . Bigger diameters generally increase the probability of surface defects. The aspect ratio of length and diameter can be ranging from a thousand to infinity in continuous fibres. They usually occupy 30-70% of the volume of the composite and 50% of its weight. The main functions of fibres are to carry the load and provide stiffness, strength, thermal stability and other structural properties to the FRP. To perform these functions, the fibres in FRP composites must have high modulus of elasticity, high ultimate strength, low variation of strength among fibres, high stability of their strength during handling and high uniformity of diameter and surface dimension among fibres (Potyrala, 2011).

Basically, there are two forms of reinforcement: rovings and fabrics (Potyrala, 2011). Roving as a one-dimensional reinforcement of polymer composites and it includes: (i) *smooth roving* - bundle of filaments arranged longitudinally in a free manner; (ii) *interlaced roving* - bundle of filaments arranged longitudinally with elementary fibres interlaced in a loop to mechanically connect neighbouring roving; (iii) *tangled roving* - bundle of filaments arranged longitudinally, interlaced mutually in order to provide better co-operation of the neighbouring filaments in a single roving; (iv) *stapled fibres* - short filaments made, for example, by cutting the smooth roving; (v) *minced fibres* - very short filaments obtained by milling and sifting stapled fibres. In order to strengthen the surface

elements in more than one direction of reinforcement (although unidirectional surface reinforcement is also produced), the following forms are applied; (a)-*smooth roving fabrics* - fabrics made of interlaced roving; (b) *interlaced roving fabrics* - interlaced roving connect neighbouring fabrics; and (c)-*mats* - made of discontinuous, random fibres.

The type of fibres used as the reinforcement is the basics for classification of FRP composites. There are three types of fibres dominating civil engineering industry: glass, carbon and aramid fibres (Potyrala, 2011). The reinforcement can be described as fibrous or particulate. The fibres are continuous (long fibre) or discontinuous (short fibre). The arrangement and orientation of these fibres determine the mechanical properties of the composite. The principal fibres used in CFRP strengthening materials are *carbon*. Carbon fibres are characterized by both high strength and stiffness, relatively low coefficient of thermal expansion and high electrical conductivity. The properties of CFRP are as follows (Gopal, 2010): (i) alkali and corrosion resistant; (ii) low thermal conductivity; (iii) high strength to weight ratio; (iv)

available in sheets and rolls of long lengths, thereby, requiring very few joints, avoiding laps and splices, and its transportation is also very easy; (v) does not require heavy equipment during construction; (vi) short curing time. Therefore, the application takes a shorter time. This reduces the project duration and down time of the structure to a great extent; and (vii) application of CFRP does not require bulky and dusty materials in a large quantity; therefore, the site remains tidier; (viii) possess high ultimate strain; therefore, they offer ductility to the structure and are very suitable for earthquake resistant applications; (ix) high fatigue resistance; so they are not easily degradable, which easily alleviates the requirement of frequent maintenance; (x) bad conductor of electricity and non-magnetic; (xi) due to the lightweight of prefabricated components in the CFRP composite, they can easily be transported. Thus, this encourages prefabricated construction; reduce site erection time, labour cost and capital investment requirement. Table 1 shows the mechanical properties of FRP including CFRP.

TABLE 1: Mechanical Properties Of Cfrp

Material	Tensile modulus (MSI)	Tensile strength (KSI)	Specific tensile modulus (MSI)	Specific tensile strength (KSI)	Specific gravity	Density (lbs./in
CFRP	33	580	18.3	322	1.8	0.65
Carbon epoxy composite	17	275	11	178	1.54	0.56
High carb. Steel 1090	30	122	3.8	15.6	7.82	0.274
Titanium	15	170	3.25	36	4.62	0.162
Aramid fibre	18	400	12.5	278	1.44	0.052

Critical Load of CFRP-strengthened Timber Columns

Consider a simply-supported CFRP-strengthened timber column subjected to an axial compression force, P , at this point, it has a differential representation as, $q = 0$; and the nonlinear governing equations become (Yu et al., 2011):

$$\begin{cases} (EI)e \frac{d}{dx} \left(\left(1 - \frac{F_N}{(EA)e} \right) \frac{d^3 w}{dx^3} \right) - F_N \frac{d^2 w}{dx^2} = 0 \\ \frac{dF_N}{dx} + (EI)e \frac{d^3 w}{dx^3} \frac{d^2 w}{dx^2} = 0 \\ \begin{cases} w = \frac{d^2 w}{dx^2} = 0, & x = 0, L, \\ F_N + (EI)e \frac{d^3 w}{dx^3} \frac{dw}{dx} = -P, \end{cases} \end{cases}$$

Obviously, the boundary value problem (Equations (1) and (2)) has a trivial zero solution

$$\begin{cases} w(x) = 0 \\ F_{N0}(x) = -P \\ (EI)e \left(1 + \frac{P}{(EA)e} \right) \frac{d^4 w'}{dx^4} + P \frac{d^2 w'}{dx^2} = 0, \\ \frac{dF_{N'}}{dx} = 0, \begin{cases} w' = \frac{d^2 w'}{dx^2} = 0, & x = 0, L, \\ F_{N=0}, & x=L \end{cases} \end{cases}$$

With standard the method for determining the critical load of a slender columns, the minimum Eigenvalue of this boundary value is (Yu et al., 2011)

$$P_{Cr} = \frac{\pi^2 (EI)e}{L^2} \frac{1}{1 - \frac{\pi^2 (EI)e}{(EA)e L^2}}$$

and the corresponding eigen function is

$$w'(x) = w_o(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) F_{N'} = 0$$

The Equation (6) is the critical load of the simply-supported FRP-strengthened timber column. If the compression deformation due to the axial force is neglected; that is, $(EA)e \rightarrow \infty$, the critical load (Equation (6)) reduces to a classical critical load of the simply-supported CFRP-strengthened timber column, that is,

$$P'_{Cr} = \frac{\pi^2(EI)e}{L^2}$$

For the L^2 buckling mode (Equation (8)), the reinforcement layer A_t on the top surface of the column is in compression, and the reinforcement layer A_b on the bottom surface is in tension. Let the structures of the two layers be the same, i.e., $h_t = h_b$, the thickness of the CFRP sheet be h_{FRP} , the tensile modulus be E_{CFRP} , and neglecting the compressive modulus, if the elastic modulus of the cohesive glue is E_G , then the equivalent elastic moduli of the two reinforcement layers are as in Equation (9):

$$\begin{cases} E_t = E_G(1 - \eta) \\ E_b = E_G(1 - \eta) + E_{CFRP}\eta \end{cases}$$

the percentage of the CFRP (η) in the reinforcement layer is given as:

$$\eta = \frac{h_{CFRP}}{h_t}$$

and therefore the thickness of CFRP is given as:

$$h_{CFRP} = h_t \eta$$

METHODOLOGY

Box Complex Method for Constrained Problems

The Box's complex method is a zero-order method for solving problems with bounds on variables and inequality constraints. The problem is of the form;

$$\text{Minimize} \quad f(x)$$

$$\text{Subject to} \quad g_j(x) \leq 0$$

$$l_i \leq x_i \leq u_i \quad i = 1 \text{ to } n$$

If there are no bounds on variables, initial bounds may be set with a wide margin. An equality constraint can be considered if it can be used to express one of the variables explicitly in terms of the others. The main assumption in the Box method is that the inequality constraints define a convex region. Convexity implies that given any two feasible points, every point along the line joining the two points is also feasible. The user must provide an initial feasible point.

Generation of a complex of k ($= 2n$) points

Let the initial feasible point be $\mathbf{x}^1$. We now proceed to generate a complex of k ($= 2n$) corner points. We know that $n + 1$ points define a simplex in an n -dimensional space. The convex hull of $n + 2$ or more points is conveniently referred to as a complex. Let us consider the stage when $i - 1$ feasible points have been generated and that their mean called center is $\mathbf{x}^c$. The next point $\mathbf{x}^i$ in the set is generated as follows. We generate a random number r in the range 0 to 1 for each of the directions $j = 1$ to n . The j^{th} component is defined using the relationship

$$x_j^i = l_j + r(u_j - l_j)$$

Having generated the n coordinates, the feasibility is checked. If the point is not feasible, $\mathbf{x}^i$ is updated using the bisection step

$$\mathbf{x}^i = 0.5(\mathbf{x}^i + \mathbf{x}^c)$$

Until $\mathbf{x}^i$ is feasible, convexity assumption of the feasible region guarantees that we get a feasible point. Then, the center $\mathbf{x}^c$ is updated to add the new point to the accepted set using

$$\mathbf{x}^c = \mathbf{x}^c + \frac{(\mathbf{x}^i - \mathbf{x}^c)}{i}$$

The process is continued until all k points of the complex are generated. We then determine the function values at the k points of the complex. The next steps of the algorithm are given (Belengudu and Chadrapatla, 1999).

1. The point $\mathbf{x}^H$ with the highest function values f_H is identified. The centroid $\mathbf{x}^0$ of the other $k - 1$ points is calculated using

$$\mathbf{x}^0 = \mathbf{x}^c + \frac{(\mathbf{x}^c - \mathbf{x}^H)}{k-1}$$

The reflection $\mathbf{x}^R$ of the highest point is calculated with respect to the centroid point $\mathbf{x}^0$ using the reflection parameter α (> 1). Box suggests a value of 1.3.

$$\mathbf{x}^R = \mathbf{x}^0 + \alpha(\mathbf{x}^0 - \mathbf{x}^H)$$

A check for the feasibility of $\mathbf{x}^R$ is conducted. If l_j is violated we set $\alpha = (l_j + 1e^{-6} - x_j^0) / (x_j^0 - x_j^H)$ or if u_j is violated we set $\alpha = (u_j - 1e^{-6} - x_j^0) / (x_j^0 - x_j^H)$ and $\mathbf{x}^R$ is evaluated again. If g is violated, the bisection step $\mathbf{x}^R = 0.5(\mathbf{x}^R + \mathbf{x}^0)$ is repeated until a feasible point is obtained. The function value f_R is now evaluated.

2. If $f_R > f_H$, then the bisection step $\mathbf{x}^R = 0.5(\mathbf{x}^R + \mathbf{x}^0)$ is repeated. If $f_R < f_H$ is not achieved after a set number of bisections, bisection is carried

out as $\mathbf{x}^R = 0.5 (\mathbf{x}^R + \mathbf{x}^L)$ with respect the lowest point until $f_R < f_H$. $\mathbf{x}^H$ is replaced by $\mathbf{x}^R$.

3. Check for convergence is made using the standard deviation σ or the variance σ^2 of the function value over the complex or the simple entity $f_H - f_{min}$. If this is less than a small preselected value ε , the convergence is achieved and the process stopped. If not, go to step 1.

Box's method is a convenient method for a quick solution of problems with inequality constraints. However, ensuring the convexity of the feasible region is the key.

3.1.2 Determination of Variables and Equations

Consider the column to be circular made up of built up glulam and wrapped externally rounded with CFRP of a suitable thickness

For a pin ended column

$$l_{eff} = 1.0l$$

Buckling load of column

$$P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}$$

Diameter of the glulam timber is determined by subtracting twice of the CFRP thickness

From the total diameter of the composite column, that is;

$$D_{GL} = D_{total} - 2t_{CFRP}$$

$$\text{Also } D_{total} = D_{GL} + 2t_{CFRP}$$

Moment of inertia of glulam

$$I_{GL} = \frac{\pi(D_{total} - 2t_{CFRP})^4}{64} = 0.016 \pi(D_{total} - 2t_{CFRP})^4$$

Moment of inertia of CFRP

$$I_{CFRP} = \frac{\pi(D_{total}^4 - D_{GL}^4)}{64} \quad (22)$$

$$I_{CFRP} = 0.016 \pi(D_{total}^4 - D_{GL}^4) = 0.25 \pi t_{CFRP}^4$$

Total moment of inertia of CFRP laminated column,

$$I_{total} = I_{GL} + mI_{CFRP},$$

where, m = modular ratio = $\frac{E_{CFRP}}{E_{GL}}$

Thus,

$$I_{total} = 0.016 \pi(D_{total} - 2t_{CFRP})^4 + 0.25 m \pi t_{CFRP}^4$$

But, E_{CFRP} is the modulus of elasticity of CFRP = 173 N/mm² and E_{GL} is the modulus of elasticity of glulam = 300 N/mm² and m is determined as 0.58. Also,

$$E_{total} = E_{GL} + mE_{CFRP} = 300 + 0.58(173) = 400.34 \text{ N/mm}^2$$

The buckling load is determined by substituting Equations (25) and (26) in Equation (18) which would give

$$P_{cr} = \frac{400.34 \pi^2 (0.016 \pi(D_{total} - 2t_{CFRP})^4 + 0.25 m \pi t_{CFRP}^4)}{1.0L^2}$$

$$P_{cr} = \frac{400.34 \pi^2 (0.05(D_{total} - 2t_{CFRP})^4 + 0.029 t_{CFRP}^4)}{1.0L^2} = \frac{197(D_{total} - 2t_{CFRP})^4 + 1778 t_{CFRP}^4}{1.0L^2}$$

Limit State Equation

The limit state equation in this study is given as:

$$Y = P_{cr} - F_{c90}$$

where, P_{cr} is the buckling load and F_{c90} is the safe load of the glulam that is the compressive load of glulam for the four strength classes in Eurocode 5 (2005).

Determination of Safe Load for Glulam

For the four strength classes of glulam, Table 2 shows their properties. Tables 2 and 3 show the strength classes of glulam and their corresponding compressive strength.

RESULTS AND DISCUSSION

The Figures 1 to 5 indicate the relationship of CFRP thickness, effective length and size of columns to the minimum or objective function. It can be clearly seen that a minimum CFRP thickness of 4mm, with a 200mm diameter of timber which is pinned in a column length of 5m is required to develop the critical safety performance of the column in combined axial and flexure load.

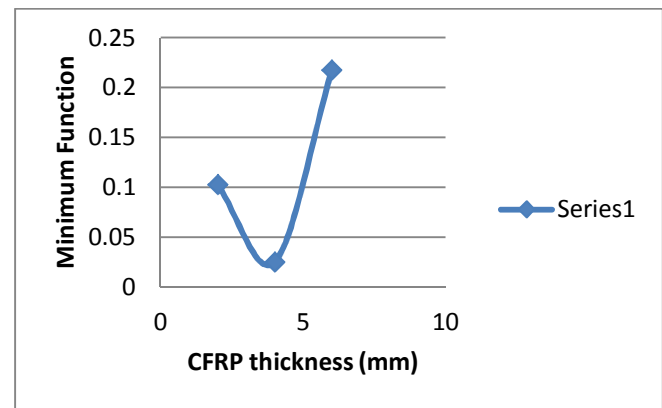


Fig 1. Relationship of CFRP thicknesses with minimum function.

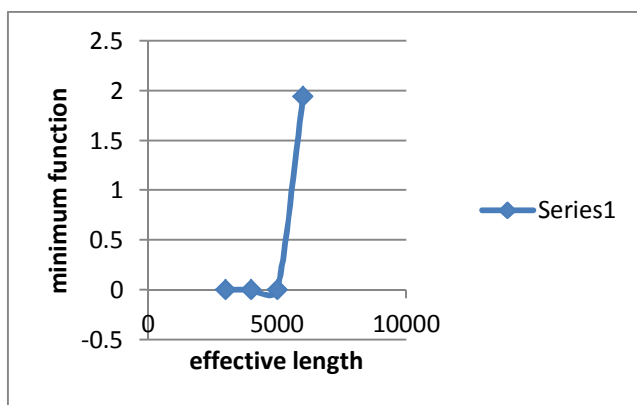


Fig 2. Relationship of effective column lengths with minimum function.

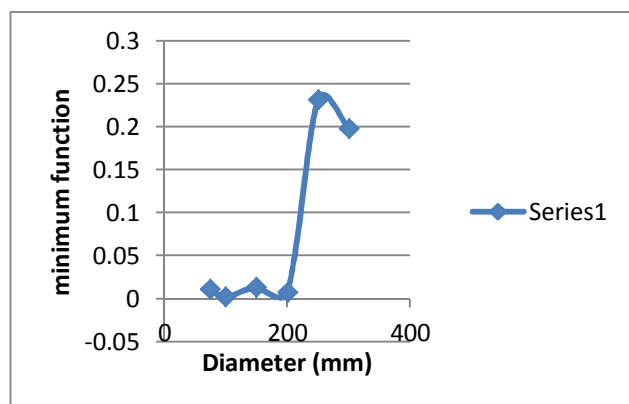


Fig 3. Relationship of columns diameters with minimum function.

CONCLUSION AND RECOMMENDATIONS

Conclusion

The thickness optimization of CFRP laminated built-up circular timber columns subjected to compression and bending loads is carried out using Box's method for complex constraint problems with the use of the Eurocode 5 (2005). Some design variables such as the diameter of column, thickness of the CFRP, effective lengths of columns were considered in the analysis.

From the foregoing variables, it was observed that for some increases in CFRP thickness there were increases in the value of the minimum function. Thus, it can be concluded that the minimum function for a column length (for example, 6.0m) and pin ended circular glulam column, wrapped externally with CFRP laminates increases as the CFRP thickness increases up to an optimum value for structural safety.

Thus the column is safer in compression at both ends at a modulus of elasticity of 400.34N/mm^2 , thickness of CFRP is 2mm minimum and 6mm

maximum, diameter of column 300mm, length of column 6000mm. Also, the column is safer in flexure,

at a modulus of elasticity of 400.34N/mm^2 , diameter of column of 300mm and 4mm thickness of CFRP.

Recommendations

Based on the results obtained and observations made in this study, the analysis using the Box method for evaluating the thickness requirement by employing the minimum function and constraint values of the structural members is recommended for further use in the examination of other possible load combinations in order to find safe and economical options for realistic designs of this type of structures. Also, it is clear that Eurocode 5 (2005) is proved to provide suitable and convenient design formulations and criteria; thus it will enable design engineers and other allied professionals to develop much confidence and satisfaction in its use.

However, due to structures inability to completely be free from the possibility of failure, design loads must therefore be in such a manner as to suitably fit the risk so that the goal of design for the required structural safety margin is not defeated at all.

REFERENCES

- Zhang, Q. P. and Chen, J. (2009) Development strategy of modern wooden architecture in China. *Science in China, Series E*, 39(5), 874–877.
- Bulleit, W. M., Sandberg, L. B., and Woods, G. J. (1989) Steel-reinforced glue laminated timber. *Journal of Structural Engineering, ASCE*, 115(2), 433–444.
- Sliker, A. (1962) Reinforced wood laminated beams. *Forest Products Journal*, 12(1), 91–96.
- Zhao, X. L. and Zhang, L. (2007) State-of-the-art review on FRP strengthened steel structures. *Engineering Structures*, 29(8), 1808–1823.
- Waldron, P., Byars, E. A., and Deijke, V. (2001) Durability of FRP in concrete: a state of the art. *Proceedings of the International Workshop on: Composites in Construction*, American Society of Civil Engineers, Capri, Italy.
- Duan, S. C. and Xu, Q. J. (2005) State-of-the-art review of composite structure of FRP and concrete (in Chinese). *Journal of Hydroelectric Engineering*, 24(5), 55–59.
- Plevris, N. and Triantafillou, T. (1992) FRP-reinforced wood as structural material. *Journal of Materials in Civil Engineering, ASCE*, 4(3), 300–317.
- Triantafillou, T. and Deskovic, N. (1992) Prestressed FRP sheets as external reinforcement of wood members. *Journal of Structural Engineering, ASCE*, 118(5), 1270–1284.
- Yang, H. F., Liu, W. Q., Shao, J. S., and Zhou, Z. H. (2008) Study on flexural behaviour of timber beams strengthened with GRP (in Chinese). *Journal of Building Materials*, 11(5), 591–597.

- Gomez, S. and Svecova, D. (2008) Behaviour of split timber stringers reinforced with external GFRP sheets. *Journal of Composites for Construction*, 12(2), 202–211.
- Gezer, H. and Aydemir, B. (2010) The effect of the wrapped carbon fiber reinforced polymer material on fir and pine woods. *Materials and Design*, 31(7), 3564–3567.
- Kim, K. J. and Harries, K. A. (2010) Modelling of timber beams strengthened with various CFRP composites. *Engineering Structures*, 32(10), 3225–3234.
- Ma, J. X., Hu, P., Jiang, X. M., and Hu, M. (2005) Experimental research on axial compressive behaviours for timber column strengthened with CFRP (in Chinese). *Industrial Construction*, 35(8), 40–44.
- Zhou, Z. H. and Liu, W. Q. (2006) Experimental study on timber columns strengthened by CFRP subjected to axial compression. *Earthquake Resistant Engineering and Retrofitting*, 28(3), 44–48.