

 Columban J. Life Sci.	Vol. 17	No. 1	127-135	2015
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EFFECTS OF MATERIAL COMPOSITION ON NATURAL FREQUENCIES OF TWO DIMENSIONAL FUNCTIONALLY GRADED ANNULAR PLATE

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Received 18 May 2015

Accepted 29 July 2015

ABSTRACT

This paper investigates free vibration of two-dimensional functionally graded annular plates (2D-FGP). The theoretical formulations are based on the three dimensional elasticity theory with small strain assumption. A two dimensional general differential quadrature method (2D-GDQ) based semi analytical approach is used to discretize partial differential equations. The material properties are assumed to be changed continuously along thickness and radial directions simultaneously, which can be varied according to exponential distribution. The effects of material graded index, and mechanical boundary conditions on the frequency parameters of the 2D-FG annular plate are evaluated in detail. The verification of the results is carried out with papers from the literature. It is apperceived that, high convergence rate for proposed 2D-GDQ approach is achieved compared with other numerical methods.

Keywords: 2D-FGM, 2D-GDQ, Free Vibrations, Annular Plate.

INTRODUCTION

In the last decade, functionally graded materials (FGMs) were employed in various engineering applications and were first reported in 1984 by Japanese material scientists (Brush and Almorph, 1975). FGMs are heterogeneous materials in which, thermo-mechanical properties vary continuously and generally as a function of position throughout the material. A typical FGM is created by smoothly changing the volume fraction of its materials constituents.

This new type of materials can be applied to avoid interfacial stress concentration appeared in laminated structures, and therefore FGMs propose great promise in applications where the working conditions are severe (Finot and Suresh, 1996) including spacecraft heat shields, heat exchanger tubes, plasma facings for fusion reactors, engine components and high-power electrical contacts or even magnets.

Prakash and Ganapathi (2006) investigated asymmetric free vibration behaviors and thermo-elastic stability of functionally graded circular plates based on the first-order shear deformation theory using finite element method. Material properties were supposed graded in the thickness direction according to simple power law distribution. For the numerical illustrations, aluminum/alumina was considered as functionally graded material. Eraslan and Akis (2006) obtained the closed form solution of functionally graded rotating solid shaft and rotating solid disks under generalized plane strain and plane stress assumptions, respectively. Efraim and Eisenberger (2007) investigated FGM plate with the material properties varying smoothly through the thickness of the plate. The equations of motion including the effect of shear deformations using the first-order shear deformation theory (FSDT) were derived and solved exactly for various arrangements of boundary

conditions. The solution was obtained by using the exact element method. Exact vibration frequencies and modes were given for several instances for the first time. Nie and Zhong (2007) studied three – dimensional vibration of functionally graded circular plates using semi-analytical approach. Dynamic analysis of multi-directional functionally graded annular plates was obtained using a semi-analytical numerical method titled the state space-based differential quadrature method. Based on the three-dimensional elastic theory and assuming the material properties having an exponent-law variation along the thickness, radial direction or both directions, the frequency equations of free vibration of multi directional functionally graded annular plates were obtained under different boundary conditions. Then, free vibration of functionally graded annular plates was investigated for different variations of material properties along the thickness, radial direction and both directions, respectively. The effect of the material property graded variations on the dynamic behavior is also investigated.

Dong (2008) studied three-dimensional free vibration of functionally graded annular plates with various boundary conditions using the Chebyshev–Ritz method, in which a set of duplicate Chebyshev polynomial series multiplied by the boundary function satisfying the boundary conditions were employed as the trial functions of the displacement components. Two types of variations of material properties in the thickness direction of the plates were considered.

Three-dimensional free vibration analyses of functionally graded annular plates using the Chebyshev-Ritz method were developed Based on the three-dimensional theory of elasticity.

Yas and Tahounh (2012) studied the free vibration of functionally graded annular plates on elastic foundations, using the differential quadrature method for various boundary conditions.

The foundation was defined by the Pasternak or two-parameter model. A semi-analytical method composed of differential quadrature method (DQM) and series solution is employed to solve the equations of motions. The material properties are varied continuously through the thickness of the plate.

Hosseini-Hashemi et al (2011) presented a solution to investigate the free vibration analysis of radially functionally graded circular and annular sector thin plates of variable thickness based on the classical plate theory (CPT) through using the differential quadrature method (DQM). The three-dimensional free vibration analysis of FG plates on elastic foundations with various boundary conditions was considered.

The material properties were supposed to be graded in the thickness direction. The equations of motion subjected to the related boundary conditions were solved by using the differential quadrature method (DQM). A detailed parametric study was carried out to highlight the effects of thickness ratios, material property graded indexes, coefficients of elastic foundations and boundary conditions on the vibration frequencies of FG thick plats.

Jodaei et al (2012) presented three-dimensional analysis of functionally graded annular plates through using state-space based differential quadrature method (SSDQM) and comparative behavior modeling by artificial neural network (ANN) for various boundary conditions.

The material properties were supposed to have an exponent-law variation along the thickness. A semi-analytical method which makes use of state-space method in thickness direction and one-dimensional differential quadrature method in radial direction was used to achieve the vibration frequencies. The state variables include a combination of three displacement parameters and three stress parameters. Once the semi-analytical approach was validated, an optimal ANN was selected, was trained and tested by the achieved numerical results.

Eventually the results of SSDQM and ANN were compared and the influence of thickness of the annular plate, material property graded index and circumferential wavenumber on the non-dimensional natural frequency of annular functionally graded material (FGM) plates with different boundary conditions were investigated. Hosseini-Hashemi et al (2010) presented for the first time, exact closed-form for frequency equations as well as transverse displacement of thick circular plates with free, soft simply supported, hard simply supported and

clamped boundary conditions based on Reddy's third-order shear deformation theory. Hamiltonian and minimum potential energy principles were used to obtain the equations of dynamic equilibrium and natural boundary conditions of the plate.

To the best of author's knowledge, the 2D generalized quadrature method has not been yet employed to study free vibration of 2D functionally graded annular plates.

Therefore, the aim of this paper is to investigate free frequency of mentioned 2D-FG annular plate. It is supposed that plate rests on different boundary conditions namely simply-simply, clamped-clamped, simply-clamped as well as free clamped on the inner edge and outer edge respectively. The equations of motion are obtained according to 3D-elasticity theory which ultimately, is reduced to the coupled differential equations.

Thereupon, semi-analytical formulation base on two dimensional GDQ approach is derived for the considered set of differential equation which result to determine frequency parameters. Convergence of the 2D-GDQ is investigated with respect to other numerical methods and results are compared with the previously published papers from the literature.

BASIC EQUATIONS

Governing equations

The equations of motion in three dimensional cylindrical coordinates system have to be formulated, in order to employ so called numerical procedure 2D-GDQ. For this purpose, it is supposed that the 2D-FG annular plate having the inner radial r_i , outer radial r_o and thickness h , with respect to the cylindrical coordinates (r, θ, z) attached on the bottom of the plate (Fig.1).

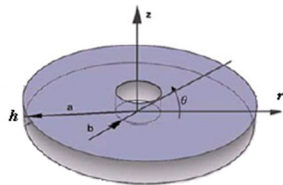


Fig. 1. Geometry and coordinate system of annular plate

In the form of cylindrical coordinates system, the linear strain-displacement relations are employed as following.

$$\begin{aligned} \varepsilon_r &= \frac{\partial U_r}{\partial r}, \varepsilon_\theta = \frac{U_r}{r} + \frac{1}{r} \frac{\partial U_\theta}{\partial \theta}, \varepsilon_z = \frac{\partial U_z}{\partial z} \\ \gamma_{r\theta} &= \frac{1}{r} \frac{\partial U_r}{\partial \theta} + \frac{\partial U_\theta}{\partial r} - \frac{U_\theta}{r}, \gamma_{rz} = \frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r}, \gamma_{z\theta} = \frac{\partial U_\theta}{\partial z} + \frac{1}{r} \frac{\partial U_z}{\partial \theta} \end{aligned} \quad (1)$$

Where U_r, U_θ and U_z are displacement components toward radial, circumferential and thickness direction and $\varepsilon_r, \varepsilon_\theta, \varepsilon_z$ are corresponding axial strain components and $\gamma_{r\theta}, \gamma_{rz}, \gamma_{z\theta}$ are in-plane and out of plane shear strain component respectively. In the absence of body forces, the equations of motion are obtained according to Eq. (2):

$$\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} &= \rho \frac{\partial^2 u_r}{\partial t^2} \\ \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \sigma_{z\theta}}{\partial z} + \frac{2\sigma_{r\theta}}{r} &= \rho \frac{\partial^2 u_\theta}{\partial t^2} \\ \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\sigma_{rz}}{r} &= \rho \frac{\partial^2 u_z}{\partial t^2} \end{aligned} \quad (2)$$

Where σ_r, σ_θ and σ_z are axial stress components and $\sigma_{r\theta}, \sigma_{rz}, \sigma_{z\theta}$ are shear stress components. Apparently, ρ denotes material density and t is time.

The stress-strain relations for the mentioned isotropic FGM are obtained according to generalized Hook's law as follow:

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \sigma_{z\theta} \\ \sigma_{rz} \\ \sigma_{r\theta} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{z\theta} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} \quad (3)$$

Where C_{ij} denotes compliance for isotropic FGM.

The displacement fields U_r, U_θ, U_z for free vibration of 2D-FG annular plates are represented as unknown functions along the radial and thickness directions and known trigonometric functions along the circumferential direction as below:

$$\begin{aligned} U_r(r, \theta, z, t) &= u_{rm}(r, z) \cos(m\theta) e^{i\omega t} \\ U_\theta(r, \theta, z, t) &= u_{\theta m}(r, z) \sin(m\theta) e^{i\omega t} \\ U_z(r, \theta, z, t) &= u_{zm}(r, z) \cos(m\theta) e^{i\omega t} \end{aligned} \quad (4)$$

where m denotes the circumferential wave number which could specifies any positive integer value ($m=0,1,\dots,\infty$). ω is the natural frequency parameter and i is the imaginary number. It is apparent that if

$m=0$, axisymmetric vibration is obtained since the effects of nonlinearity are ignored.

Combining strain-displacement relations Eq.(1) into Hook's law Eq.(3) and substituting into equations of motion Eq. (2) and then using the trigonometric functions Eq.(4) we can write,

$$\begin{aligned} & \frac{\partial C_{11}}{\partial r} \frac{\partial u_r}{\partial r} + C_{11} \frac{\partial^2 u_r}{\partial r^2} + \frac{\partial C_{12}}{\partial r} \left(\frac{u_r}{r} + \frac{1}{r} u_{\theta m} \right) + \frac{\partial C_{13}}{\partial r} \frac{\partial u_z}{\partial z} + C_{13} \frac{\partial^2 u_z}{\partial r \partial z} + \frac{C_{11} - C_{12}}{r} \frac{\partial u_r}{\partial r} + \\ & C_{12} \left(\frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial u_{\theta}}{\partial r} m \right) - C_{12} \left(\frac{u_r}{r^2} + \frac{1}{r^2} u_{\theta m} \right) + \frac{C_{66}}{r} \left(-\frac{u_{\theta}}{r} m + \frac{\partial u_{\theta}}{\partial r} m - \frac{1}{r} u_r m^2 \right) + \end{aligned} \quad (5)$$

$$\begin{aligned} & \frac{\partial C_{55}}{\partial z} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) + C_{55} \left(\frac{\partial^2 u_r}{\partial z^2} + \frac{\partial^2 u_z}{\partial r \partial z} \right) + \frac{C_{12} - C_{22}}{r} \left(\frac{u_r}{r} + \frac{1}{r} u_{\theta m} \right) + \frac{C_{13} - C_{23}}{r} \frac{\partial u_z}{\partial z} = -\rho \omega^2 u_r, \\ & \frac{\partial C_{66}}{\partial r} \left(-\frac{u_{\theta}}{r} + \frac{\partial u_{\theta}}{\partial r} - \frac{1}{r} u_r m \right) + C_{66} \left(-\frac{1}{r} \frac{\partial u_r}{\partial r} m - \frac{1}{r} \frac{\partial u_{\theta}}{\partial r} + \frac{u_{\theta}}{r^2} + \frac{\partial^2 u_{\theta}}{\partial r^2} + \frac{1}{r^2} u_r m \right) - \end{aligned} \quad (6)$$

$$\begin{aligned} & \frac{C_{12}}{r} \frac{\partial u_r}{\partial r} m - \frac{C_{23}}{r} \frac{\partial u_z}{\partial z} m - \frac{C_{22}}{r^2} (u_r m + u_{\theta} m^2) + \frac{\partial C_{44}}{\partial z} \left(\frac{\partial u_{\theta}}{\partial z} - \frac{1}{r} u_z m \right) + \\ & C_{44} \left(\frac{\partial^2 u_{\theta}}{\partial z^2} - \frac{1}{r} \frac{\partial u_z}{\partial z} m \right) - \frac{2C_{66}}{r^2} u_r m + \frac{2C_{66}}{r} \frac{\partial u_{\theta}}{\partial r} - \frac{2C_{66}}{r^2} u_{\theta} = -\rho \omega^2 u_{\theta} \end{aligned}$$

$$\begin{aligned} & \frac{\partial C_{55}}{\partial r} \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) + C_{55} \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{\partial^2 u_r}{\partial r \partial z} \right) + \frac{C_{44}}{r} \frac{\partial u_{\theta}}{\partial z} m - \frac{C_{44}}{r^2} u_z m^2 + \frac{\partial C_{13}}{\partial z} \frac{\partial u_r}{\partial r} + \\ & C_{13} \frac{\partial^2 u_r}{\partial r \partial z} + \frac{\partial C_{23}}{\partial z} \left(\frac{u_r}{r} + \frac{1}{r} u_{\theta m} \right) + C_{23} \left(\frac{1}{r} \frac{\partial u_r}{\partial z} + \frac{1}{r} \frac{\partial u_{\theta}}{\partial z} m \right) + \frac{\partial C_{33}}{\partial z} \frac{\partial u_z}{\partial z} + C_{33} \frac{\partial^2 u_z}{\partial z^2} + \\ & \frac{C_{55}}{r} \frac{\partial u_r}{\partial z} + \frac{C_{55}}{r} \frac{\partial u_z}{\partial r} = -\rho \omega^2 u_z \end{aligned} \quad (7)$$

Relations (5)-(7) denote 3D elasticity solution for equations of motion in terms of displacement components and must be solved for known compliances and assumptive displacement fields have been supposed in Eq.4. Then it must be solved according to one of existing numerical procedure. Here 2D-GDQ procedure has been considered to discretize linear set of equations Eq. (5)-(7). Precedence of 2D-GDQ approach compared with one dimensional procedure is for type of boundary condition and grids distribution toward two directions which is illustrated in the coming sections. In order to investigate free vibration of 2D-FG annular plate in different foundation, four different kinds of boundary condition are considered namely clamped-clamped (C-C), simply supported –clamped (S-C), free-clamped (F-C) and simply supported – simply supported(S-S). The boundary conditions at edges for mentioned foundations are according to below:

Clamped-clamped (C-C):

$$r = r_i, r_o \quad u_r = u_{\theta} = u_z = 0 \quad (8)$$

Simply supported – clamped (S-C):

$$\begin{aligned} r = r_i \quad u_{\theta} = u_z = \sigma_r = 0 \\ r = r_o \quad u_r = u_{\theta} = u_z = 0 \end{aligned} \quad (9)$$

Free-clamped (F-C):

$$\begin{aligned} r = r_i \quad \sigma_r = \sigma_{r\theta} = \sigma_{rz} = 0 \\ r = r_o \quad u_r = u_{\theta} = u_z = 0 \end{aligned} \quad (10)$$

Simply supported – simply supported (S-S):

$$r = r_i, r_o \quad u_{\theta} = u_z = \sigma_r = 0 \quad (11)$$

Micromechanics of 2D-FGM annular plate

A typical FGM is made of two randomly distributed isotropic constituent, i.e. a metal and a ceramic.

The macroscopic response of the composite is isotropic and the compositions of the composite vary throughout radial and thickness directions.

Such a material allows continuous variation of the material composition, changing natural frequency parameter.

One important stage in determining the micromechanical model of FG circular plate is homogenization of the composite.

Choice of the approach should be based on the gradient of gradation relative to the size of a typical representative volume element (RVE).

Here, materials distribution are employed based on the exponential law distribution. The material elastic coefficients (C_{ij}) and the mass density (ρ) are assumed to have the following exponential law distribution along the thickness and radial directions of plate as following (G. Nie , Z. Zhong, 2010):

$$\begin{aligned} C_{ij}(r, z) &= C_{ij}(0,0) \exp(\lambda_1 \frac{z}{h} + \lambda_2 \frac{r}{r_o}) \\ \rho_{ij}(r, z) &= \rho_{ij}(0,0) \exp(\lambda_1 \frac{z}{h} + \lambda_2 \frac{r}{r_o}) \end{aligned} \quad (12)$$

here $C_{ij}(0,0)$ and $\rho_{ij}(0,0)$ are the reference values at the center point of the bottom plane, λ_1 and λ_2 denote the material property graded indexes in the thickness and radial directions, respectively.

In first step, the material properties of the annular plate are assumed as the exponent-law variation in the thickness and radial directions.

Young's modulus at the center of the bottom plane is $E = 380$ (GPa) and Poisson's ratio is chosen as constant ($\nu = 0.3$).

The material density at the center of the bottom plane is $\rho = 3800$ (kg/m³).

By definition of non-dimensional parameter $\varsigma = \frac{z}{h} - \frac{1}{2}$ in thickness and $\eta = \frac{r - R}{H}$ in radial directions, density variation for different values of material property graded indexes is demonstrated (Fig. 2).

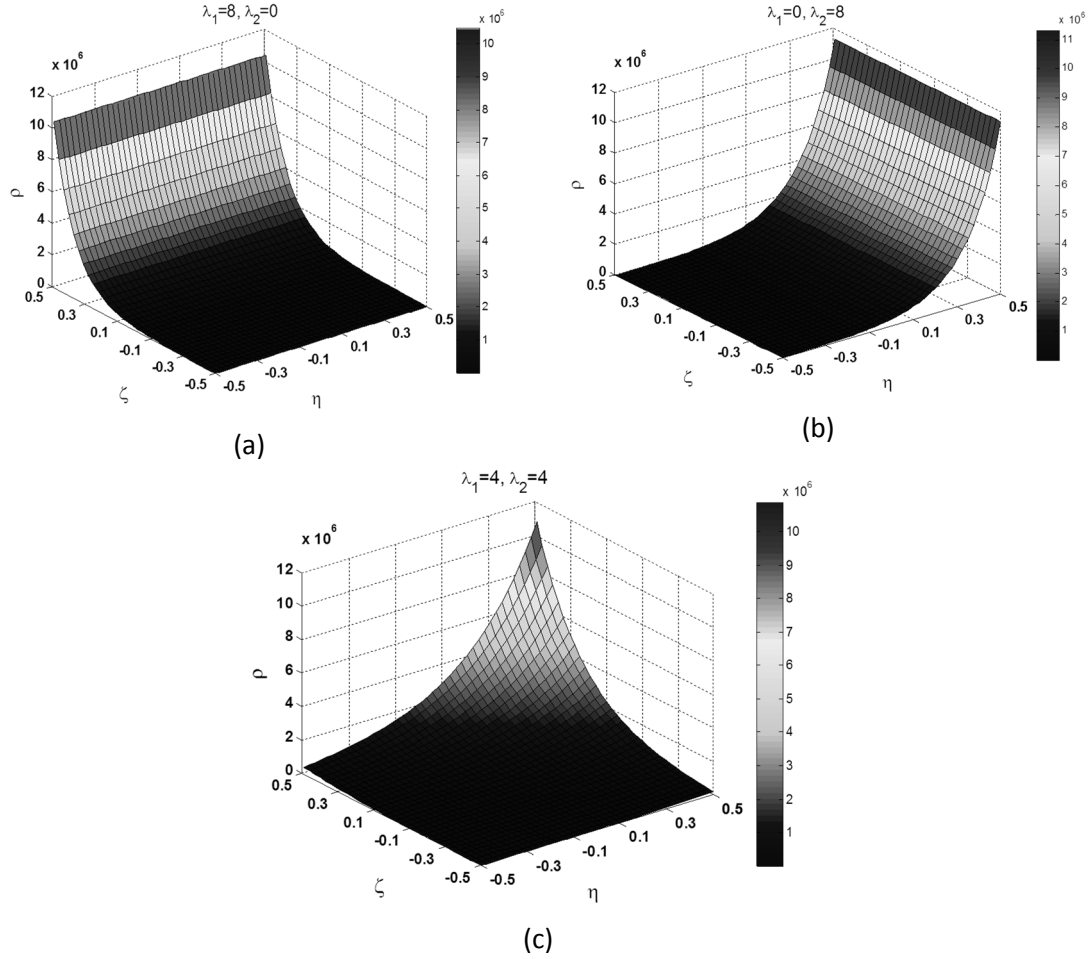


Fig. 2. Density variation for (a) $\lambda_1 = 8, \lambda_2 = 0$ (b) $\lambda_1 = 0, \lambda_2 = 8$ (c) $\lambda_1 = 4, \lambda_2 = 4$

Where R is average radius of annular plate and $H = r_o - r_i$.

SOLUTION PROCEDURE

It is difficult to solve the equations of motion analytically. Hence, one should use a numerical procedure in order to discretize partial differential equations in terms of displacement components. Then it must solve the set of discretized algebraic equation as an Eigen-value problem to find natural frequencies. Apparently, Eqs. (5)-(7) are linear set of partial differential equations of motion which ought to be solved with one of numerical solver. Here, the 2D-GDQ procedure is used to discretize the motion equations and followed by related boundary conditions. Choose of solution procedure i.e. 2D-GDQ method is one based on type of boundary

conditions and second for meshing the plate in two direction (Here grids are distributed toward material variations namely, radial and thickness direction.

The differential quadrature method (GDQ) is a numerical solution technique for initial and/or boundary value problems (Bert and Malik, 1996). It was first developed by Richard Bellman and his associates in the early 1970s (Bellman et al., 1972). The DQ method is originated from the idea of conventional integral quadrature and approximates the derivative of a function at any discrete point by a weighted linear summation of the functional values in the whole domain (Shu, 2000).

A simple and accepted choice of the grid distribution is the uniform grid spacing rule; however, it was found that non-uniform grid spacing yields results with better accuracy. Hence, in this study, the

Chebyshev–Gauss– Lobatto quadrature points are employed, that is (Bert and Malik, 1996):

$$\xi = \frac{1}{2} \left\{ 1 - \cos \left(\frac{\pi(i-1)}{N_r-1} \right) \right\}; \text{ for } i = 1, 2, \dots, N_r \text{ and } j = 1, 2, \dots, N_z$$

$$\varsigma = \frac{1}{2} \left\{ 1 - \cos \left(\frac{\pi(j-1)}{N_z-1} \right) \right\} \text{ for } i = 1, 2, \dots, N_r \text{ and } j = 1, 2, \dots, N_z \quad (13)$$

Where ξ and ς are distribution of nodes along the r and z directions, respectively. As one can observe, with this non-homogeneous distribution of nodes, by approaching to internal and external boundaries of FGM annular plate with various boundary conditions and also upper and lower boundaries with free boundary condition, density of nodes increases while it causes boundary conditions get better satisfied. By getting far from boundaries, less density in node of grid is generated. In 2D-GDQ method, in order to approximate the first-order derivatives of the function $f(r, z)$ with respect to r in a node of grid, according to Eq.(14), the summation of product of function values in all nodes in the r direction and parallel to the target node with the weight coefficients in the direction r .

$$\frac{\partial f(r, z)}{\partial r} = \sum_{l=1}^{N_r} A_{jl}^{(r)} f_{lk} \quad (14)$$

Using the 2D-GDQ method for the spatial derivatives, the discretized form of the equations of motion at each domain grid point can be obtained.

In order to carry out the eigenvalue analysis, the domain and boundary degrees of freedom are separated and in vector forms. They are denoted as $\{d\}$ and $\{b\}$, respectively. Based on this definition, the discretized form of the motion equations and the related boundary conditions take the following forms: Equations of motion:

$$[[K_{db}] \quad [K_{dd}]] \begin{Bmatrix} \{b\} \\ \{d\} \end{Bmatrix} - \omega^2 [M] \begin{Bmatrix} \{b\} \\ \{d\} \end{Bmatrix} = \{0\} \quad (15)$$

Boundary conditions:

$$[K_{bd}] \{d\} + [K_{bb}] \{b\} = \{0\} \quad (16)$$

By eliminating the boundary degrees of freedom in Eq. (15), using Eq. (16), this equation turn into:

$$([K] - \omega^2 [M]) \{d\} = \{0\} \quad (17)$$

Where $[K] = [K_{dd}] - [K_{db}][K_{bb}]^{-1}[K_{bd}]$. The above eigenvalue system of equations can be solved to find the natural frequencies and mode shapes of the annular plates.

NUMERICAL RESULTS AND DISCUSSION

Verification of results

Free vibration analysis of two dimensional functionally graded annular plates (2D-FG annular plate) was performed via general differential quadrature method (GDQ). The equations of motion have been extracted according to 3D elasticity theory. For this purpose, a 2D-FG annular plate with arbitrary dimension must be considered. All values including dimensions of the plate as well as frequency parameters are presented as non-dimensional form for ease of use. In order to ensure of accuracy of the results, comparisons have been performed from the literature. Firstly a homogenous and isotropic annular plate with clamped-clamped boundary condition is considered. In Tables.1 presented results are compared with similar ones in the literatures, namely homogenous isotropic annular plate ($\Omega = \omega R_o \sqrt{\frac{\rho}{G_m}}$). Results obtained by analysis of 2D-

GDQ are shown in Tables 1 and it is obvious that faster convergence rate for proposed 2D-GDQ procedure are obtained in comparison with Chebyshev–Ritz method (Dong, 2008) and Zhou (2003).

Table: 1. The non-dimensional frequencies for isotropic annular plates with C-C condition ($\frac{R_o}{R_i} = 2.5, \frac{h}{R_o} = 0.5$)

M	method	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
0	present	4.6582	8.9505	11.3915	14.3382	17.2782	19.2184
	Dong (2008)	4.661	8.952	11.40	14.34	17.29	19.22
	Zhou (2003)	4.660	8.950	11.40	14.34	17.28	19.22
1	present	4.7482	8.5359	9.1248	11.2232	12.7365	14.4148
	Dong (2008)	4.752	8.536	9.127	11.23	12.74	14.42
	Zhou (2003)	4.750	8.536	9.126	11.23	12.74	14.42
2	Present	5.0865	9.0963	9.7069	11.1450	13.6461	14.6436
	Dong (2008)	5.089	9.097	9.709	11.15	13.65	14.65
	Zhou (2003)	5.088	9.096	9.708	11.15	13.65	14.65
3	Present	5.7206	9.7439	10.5762	11.4287	14.7313	15.0190
	Dong (2008)	5.723	9.745	10.58	11.43	14.73	15.02
	Zhou (2003)	5.722	9.744	10.58	11.43	14.73	15.02

Hence, good agreement is achieved within the results. For this verification, the number of grids in radial and thickness directions for the 2D- GDQ is assumed $N_r = N_z = 15$.

Numerical Results

In the recent work, two dimensional GDQ procedures are considered in order to investigate natural frequency parameter of 2D-FG annular plate. Then the effects of different parameters such as BCs as well as power exponent coefficients are studied. There are several efforts in which mechanical dynamic response of FGMs have been studied, but in the case of vibration analysis of 2D-FG plate using 2D-GDQ method, lack of knowledge is perceived.

Two papers deal with free vibration analysis of homogenous and FG isotropic annular plates were presented in previous section in order to validate the results. Hence we are satisfied to results which presented in previous section. function of constituent distribution namely exponential was introduced presented in previous section, reveals functionality of material distribution with respect to the radial and thickness directions. It is assumed that the plate stands in ambience temperature with different mechanical boundary condition were introduced in section 2. In the coming section, the effect of boundary condition is investigated on the natural frequency parameters of 2D-FG annular plates with proposed 2D GDQ algorithm.

The significant difference within numerical results reveals in convergence rate. Generalized differential quadrature method due to lesser number of grids for convergence and also the excellent solutions in comparison with other numerical methods is more popular. On the other hand, in the proposed 2D-GDQ algorithm, it has been tried to mesh the plate in two direction of functionality including radial and thickness direction, respectively as it was illustrated in prior section. Meshing the plate in two directions of functionality and using 2D-GDQ procedure leads to obtain more accurate result with faster rate of convergence. 3D graphs are used, in order to observe convergence rate of the algorithm. The number of points required for convergence in radius and thickness directions is studied. Rate of convergence for the first non-dimensional natural frequencies in

terms of different numbers of nodes in radial(N_r) and thickness(N_z) directions is shown in Fig.3.

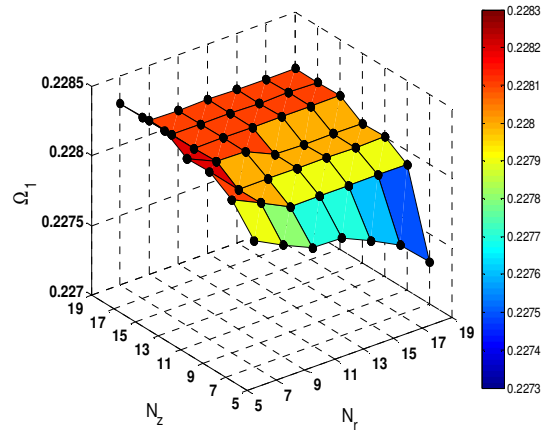


Fig. 3. Rate of convergence for the first natural frequencies $=1, h=0.2, \lambda_1 = \lambda_2 = 1 r_i$) in terms of simply supported – clamped condition($=0.2, r_o$

Flat part at Fig.3 shows the first frequencies achieve convergence for a certain number of nodes along the r and z, in other words, significant changes in response to increasing the number of nodes in the two-dimensional grid, has not been observed. For example, in Fig.3 per nine nodes in the r and 15 nodes along the z directions, converges in the first frequency.

As observed, the convergence is fast with low number of grid points along the radial and thickness directions are required. As one can observe, the solution converges with low number of grid points along the radial and thickness directions ($N_r = 9, N_z = 15$).

Variations of first non-dimensional natural frequencies versus material graded index in case of conventional FG annular plate along thickness or radial and 2D-FG annular plate in both direction, simultaneously for different boundary conditions are demonstrated in Figs.(4-6):($r_i=0.2, r_o=1, h=0.2$)

In Fig.6, it is assumed $\lambda_1 + \lambda_2 = 8$. It is observed that boundary conditions have significant effects on natural frequency parameter of FG annular plate. Figs. 4-5 depict the first non-dimensional natural frequency variations of conventional FG annular

plate versus variation of power coefficients along thickness and radial directions, under different boundary conditions, respectively.

Fig. 4 shows variation of frequency versus power coefficient for conventional FG annular plate which functionality of materials is throughout thickness direction. It is observed that frequency trends to decrease when power coefficient increases. Also, higher natural frequency is obtained for a plate with fully clamped edges. Additionally, slop of frequency decrease for a plate with fewer constraints i.e. here F-C plate is less in comparison other type of boundary condition.

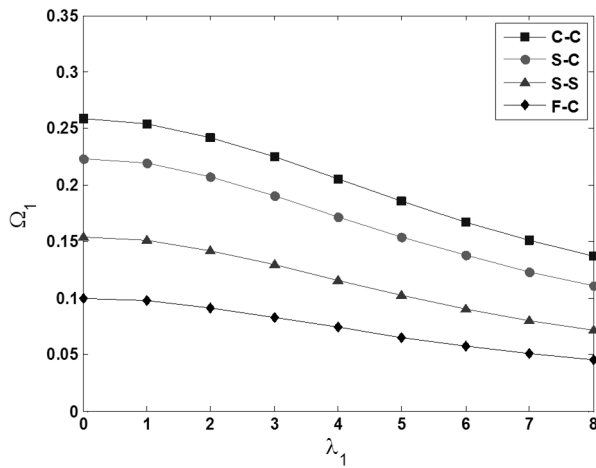


Fig. 4. Variations of first non-dimensional natural frequency parameter for 1D-FGM versus graded index in thickness direction ($\lambda_2 = 0$)

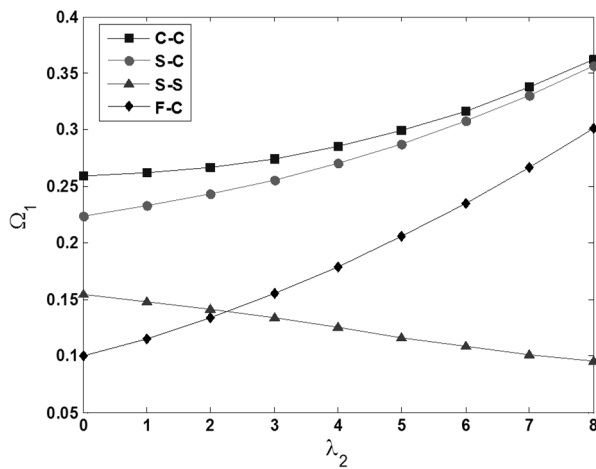


Fig. 5. Variations of first non-dimensional natural frequency parameter for 1D-FGM versus graded index in radial direction ($\lambda_1 = 0$)

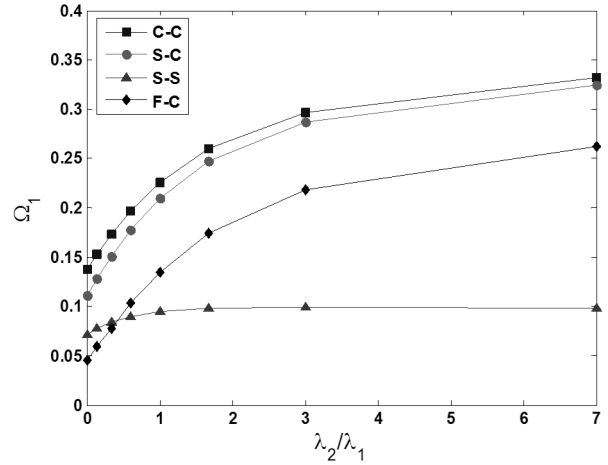


Fig. 6. Variations of first non-dimensional natural frequency parameter versus ratio of radial graded index to thickness graded index for 2D-FGM

For the case of functionally graded plate along the radial direction, the frequency variation is quietly different. As observed the first non-dimensional natural frequency of the annular plate under all boundary conditions except (S-S) increases with the increase of functionally graded index in radial direction, but in (S-S) condition, it is reduced. Fig.6 for 2D-FGM shows a same behavior for all boundary conditions, the first non-dimensional natural frequency gradually increases with the increase of the ratio of the radial graded index to thickness graded index. In Fig.5, it is also noticed for an annular plate with material property graded index in radial direction lower than 2, and in Fig.6, for 2D-FGM with material graded index ratio almost less than 0.5, the first non-dimensional natural frequency with (S-S) condition is higher than one with (F-C) condition. It is noticed for 2D-FGM with (S-S) condition, the first non-dimensional natural frequency is almost constant for material graded index ratio greater than 1.

CONCLUSIONS

The mechanical free vibration of 2D-FG annular plates was investigated by using general differential quadrature method (GDQ) based on two dimensional meshing. Three dimensional elasticity theory were employed in order to derive equations of motion for 2D-FG annular plates. The mechanical properties of 2D-FG annular plate were assumed to be temperature

independent since only mechanical vibration analysis was performed. Material constituents are distributed as function of r and z coordinates simultaneously by exponential formulation.

Semi-analytical approach composed of two-dimensional generalized differential quadrature methods (2D-GDQ) and series solution were adopted to solve the equations of the motion. The effect of functionality of material properties with respect position, boundary conditions including simply supported-clamped, clamped-clamped, free-clamped and simply supported-simply supported in inner and outer radius, respectively as well as aspect ratio were taken into account too. Results for natural frequency parameter of 2D-FG annular plate with different parameters have illustrated the fast convergence and high accuracy of the proposed approach in spite of using 2D grids. From this study, the following conclusion can be made:

- Convergence of 2D-GDQ method is fast and the results confirm well previous investigations.
- In the terms of functionally graded indexes in thickness or radial directions increase, first non-dimensional natural frequencies increase for all kinds of boundary conditions is observed, but (s-s) boundary condition for the uni-directional FGM annular plate in radial direction, shows subtractive trend.
- It results that the non-dimensional natural frequencies of thin uni-directional functionally graded along radial or thickness as well as thin 2D-FGM annular plate doesn't depend on the functionally graded indexes in thickness and radial directions.
- It results for 2D-FGM with (S-S) condition; the first non-dimensional natural frequency is almost constant for material graded index ratio greater than 1.

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